GCSE Higher Tier Mathematics

Rational Expressions

FREE SAMPLE WORKSHEET 10
Includes Full Model Answers

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Instructions

- Answer all questions.
- Your working must clearly show the complete factorisation of both the numerator and denominator before any cancellation.
- The number of marks is given in brackets [ ] at the end of each question or part question.
- Calculators should **not** be used for this worksheet.
- The final answer must be in its simplest form, achieved by cancelling all common factors.

Key Concepts: Factorising and Simplifying Rational Expressions

A **rational expression** is an algebraic fraction where the numerator and denominator are polynomials. The key to simplifying them is to find and cancel common **factors**, not common terms.

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1. The Fundamental Method

Method: Simplifying Rational Expressions

The process never changes. It is a strict, two-step procedure.

1. **Factorise Fully:** Factorise the numerator and the denominator completely. You must look for every possible factor.
2. **Cancel Common Factors:** Cancel any factor that is identical in both the numerator and the denominator.

Caution: Never Cancel Terms, Only Factors!

This is the single most common error. You cannot cancel individual terms across a plus or minus sign. The expression **must** be fully factorised first.

Example: In the expression $\frac{x^2-9}{x-3}$, it is wrong to cancel the x's.

Wrong: $\frac{\cancel{x}^{\cancel{2}}-9}{\cancel{x}-3}$ **Incorrect!**

Right: $\frac{(x-3)(x+3)}{x-3} = \boldsymbol{x+3}$ **Correct!**

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2. Your Factorising Toolkit

Pro-Tip: Remember All Your Factorising Methods

To simplify these expressions, you will need to be fluent in all forms of factorisation:

- **Common Factor:** e.g., $3x^2 - 6x = 3x(x-2)$
- **Difference of Two Squares:** e.g., $a^2 - b^2 = (a-b)(a+b)$
- **Quadratics (Monic):** e.g., $x^2 + 5x + 6 = (x+2)(x+3)$
- **Quadratics (Non-Monic):** e.g., $2x^2 - 5x - 3 = (2x+1)(x-3)$
- **Grouping:** For expressions with 4 terms, e.g., $xy+2x+3y+6 = (x+3)(y+2)$

Always look for a common factor first before trying other methods!

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3. Factoring out -1

Deeper Insight: Using -1 to Create a Common Factor

Sometimes factors look very similar but are reversed, like $(x-4)$ and $(4-x)$. They are not the same, but they are related by a factor of -1.

The key relationship is: $\boldsymbol{(b-a) = -(a-b)}$.

Example: Simplify $\frac{2x-8}{4-x}$.

$$\frac{2(x-4)}{4-x} = \frac{2(x-4)}{-1(x-4)} \quad \text{(Factor out -1 from the denominator)}$$

$$= \frac{2}{-1} = \mathbf{-2}$$

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1. Simplify the following rational expressions completely:

   (a) $\frac{3x+6}{9x+18}$ [2]

   (b) $\frac{4a-8}{2a-4}$ [2]

   (c) $\frac{p^2+pq}{5p+5q}$ [2]

   (d) $\frac{6m-3}{1-2m}$ [2]

Total: [8]

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2. Simplify the following rational expressions completely:

(a) $\frac{x^2+x-6}{x+3}$ [2]

(b) $\frac{y-4}{y^2-16}$ [2]

(c) $\frac{2a^2+5a-3}{a+3}$ [2]

(d) $\frac{3p+9}{p^2+5p+6}$ [2]

(e) $\frac{k^2-8k+15}{2k-10}$ [3]

Total: [11]

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3. Simplify the following rational expressions completely:

(a) $\frac{x^2-4}{x^2+x-6}$ [3]

(b) $\frac{y^2+7y+10}{y^2+6y+5}$ [3]

(c) $\frac{a^2-8a+12}{a^2-4a-12}$ [3]

(d) $\frac{2p^2+p-1}{p^2+p}$ [3]

(e) $\frac{3m^2-7m+2}{m^2-4}$ [3]

Total: [15]

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4. Simplify the following rational expressions completely:

(a) $\frac{2x^2-8}{x^2+2x}$ [3]

(b) $\frac{xy+2y+3x+6}{y^2+3y}$ [3]

(c) $\frac{ax-bx+ay-by}{a^2-b^2}$ [3]

(d) $\frac{p^3-p}{p^2+p}$ [3]

Total: [12]

This is the end of the worksheet

1)

Reminder: The Core Method

The core method is always the same:

(a) Factorise the numerator and denominator completely.

(b) Cancel any identical factors that appear in both.

(a)

$$\frac{3x+6}{9x+18} = \frac{3(x+2)}{9(x+2)}$$

$$= \frac{\cancel{3}\cancel{(x+2)}}{\cancel{9}_3\cancel{(x+2)}} = \mathbf{\frac{1}{3}}$$

(b)

$$\frac{4a-8}{2a-4} = \frac{4(a-2)}{2(a-2)}$$

$$= \frac{\cancel{4}^2\cancel{(a-2)}}{\cancel{2}\cancel{(a-2)}} = \mathbf{2}$$

(c)

$$\frac{p^2+pq}{5p+5q} = \frac{p(p+q)}{5(p+q)}$$

$$= \boldsymbol{\frac{p}{5}}$$

(d)

Deeper Insight: Factoring Out -1

To reveal the common factor, we can factor out -1. Remember: $(b-a) = -(a-b)$.

So, $(1-2m) = -1(2m-1)$.

$$\frac{6m-3}{1-2m} = \frac{3(2m-1)}{-1(2m-1)}$$

$$= \frac{3}{-1} = \mathbf{-3}$$

2)

Pro-Tip: Use Your Factorising Toolkit

When you see a quadratic expression, remember your factorising methods. Look for patterns like the Difference of Two Squares (e.g., $y^2 - 16$ in part b) to simplify the process.

(a)

$$\frac{x^2 + x - 6}{x + 3} = \frac{(x+3)(x-2)}{x+3}$$

$$= \boldsymbol{x - 2}$$

(b)

$$\frac{y - 4}{y^2 - 16} = \frac{y - 4}{(y-4)(y+4)}$$

$$= \frac{\mathbf{1}}{\boldsymbol{y + 4}}$$

(c)

$$\frac{2a^2 + 5a - 3}{a + 3} = \frac{(2a-1)(a+3)}{a+3}$$

$$= \boldsymbol{2a - 1}$$

(d)

$$\frac{3p+9}{p^2+5p+6} = \frac{3(p+3)}{(p+2)(p+3)}$$

$$= \frac{\mathbf{3}}{\boldsymbol{p}+\mathbf{2}}$$

(e)

$$\frac{k^2-8k+15}{2k-10} = \frac{(k-3)(k-5)}{2(k-5)}$$

$$= \frac{\boldsymbol{k}-\mathbf{3}}{\mathbf{2}}$$

3)

Caution: Do Not Cancel Terms!

This is where the temptation to cancel individual terms (like the a^2 terms in part c) is strongest. You must resist! Only cancel factors, which means the expression must be fully factorised first.

(a)

$$\frac{x^2-4}{x^2+x-6} = \frac{(x-2)(x+2)}{(x+3)(x-2)}$$

$$= \frac{\boldsymbol{x}+\mathbf{2}}{\boldsymbol{x}+\mathbf{3}}$$

(b)

$$\frac{y^2+7y+10}{y^2+6y+5} = \frac{(y+2)(y+5)}{(y+1)(y+5)}$$

$$= \boldsymbol{\frac{y+2}{y+1}}$$

(c)

$$\frac{a^2-8a+12}{a^2-4a-12} = \frac{(a-2)(a-6)}{(a+2)(a-6)}$$

$$= \boldsymbol{\frac{a-2}{a+2}}$$

(d)

$$\frac{2p^2+p-1}{p^2+p} = \frac{(2p-1)(p+1)}{p(p+1)}$$

$$= \boldsymbol{\frac{2p-1}{p}}$$

(e)

$$\frac{3m^2-7m+2}{m^2-4} = \frac{(3m-1)(m-2)}{(m-2)(m+2)}$$

$$= \boldsymbol{\frac{3m-1}{m+2}}$$

4)

Pro-Tip: Look for the HCF First

These more complex expressions often require a combination of factorising techniques. Always look for the simplest method first by taking out a Highest Common Factor (HCF), before moving on to other methods like grouping or DOTS.

(a)

$$\frac{2x^2-8}{x^2+2x} = \frac{2(x^2-4)}{x(x+2)}$$

$$= \frac{2(x-2)(x+2)}{x(x+2)}$$

$$= \boldsymbol{\frac{2(x-2)}{x}}$$

(b) The numerator has four terms and can be factorised by grouping.

$$\frac{xy + 2y + 3x + 6}{y^2 + 3y} = \frac{y(x + 2) + 3(x + 2)}{y(y + 3)}$$

$$= \frac{(x + 2)(y + 3)}{y(y + 3)}$$

$$= \boldsymbol{\frac{x + 2}{y}}$$

(c) Factorise the top by grouping and the bottom using DOTS.

$$\frac{ax - bx + ay - by}{a^2 - b^2} = \frac{x(a-b) + y(a-b)}{(a-b)(a+b)}$$

$$= \frac{(a-b)(x+y)}{(a-b)(a+b)}$$

$$= \boldsymbol{\frac{x+y}{a+b}}$$

(d) Factorise the top and bottom by taking out the HCF first.

$$\frac{p^3 - p}{p^2 + p} = \frac{p(p^2-1)}{p(p+1)}$$

$$= \frac{p(p-1)(p+1)}{p(p+1)}$$

$$= \boldsymbol{p - 1}$$